Nomination for Research Award

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1 Laudatio of Edward Belbruno

Edward Belbruno was once called in the *New Scientist* one of the "top 10 most influential space thinkers" [1]. Indeed, Edward Belbruno is the man who revolutionized and is still revolutionizing our understanding of space travel. It is the work of Edward Belbruno which shows us that methods from chaos theory of dynamical systems can be fruitfully applied to mission design. Charles Conley first proposed to take advantage of the dynamics of the restricted three body problem to find low energy transit orbits from the Earth to the Moon, namely orbits which require much less fuel than the traditional orbits which are based on Keplerian motion [2]. 1986 found Edward Belbruno the first realistic low energy transit orbit from an orbit around the Earth to an orbit around the Moon [3]. Although at the beginning many people were sceptical that such a revolutionary new idea works in practice, the first Japanese lunar mission proved in a dramatic way that Edward Belbruno was right. In January 1990 the Japanese launched their first lunar mission. There were two robotic spacecrafts involved in this mission MUSES-A (later on renamed Hiten) and MUSES-B (later on renamed Hagoromo). MUSES-B was supposed to go to the Moon where MUSES-A was to stay on an orbit around the Earth as a communication relay with MUSES-B. Unfortunately, contact with MUSES-B was lost and only MUSES-A remained. Since MUSES-A was never supposed to go to the Moon it had only little fuel, too less to travel to the Moon on a standard route. On October 2, 1991, Hiten successfully arrived at the Moon after a long travel which took advantage of chaotic motion in the Earth-Moon-satellite system as well as in the Sun-Earth-satellite system using a Weak Stability Boundary transfer of Belbruno and Miller [4], which was based on the ideas of Belbruno in [3]. Since then the ideas of Edward Belbruno have been used in several space missions, like NASA's GRAIL mission in 2011 or ESA's SMART-1 mission in 2004. Enthusiastically Senator Tom Harking from Iowa presented the discoveries of Edward Belbruno to the US congress with the words [5]

> ... physically it is fascinating about how we can use the gravity of the Sun, the Moon, and the Earth to launch vehicles from here to the Moon or to Mars or beyond and use 40 percent less energy.

Edward Belbruno is as well an extremely gifted teacher. In his book “Capture Dynamics and Chaotic Motions in Celestial Mechanics“ [6] he not only gives a very readable account of celestial mechanics, dynamical systems and dynamical astronomy but as well treats his new discoveries and capture dynamics in detail, so that this book has now become a standard reference for everybody interested in capture dynamics and celestial mechanics in general. In the book “Fly Me To The Moon“ [7] Edward Belbruno tells the many ups and down in the thrilling story about the saving of the Hiten mission to a broad audience, inspiring many young researchers to ponder deeply about the wealth of interesting research questions his discoveries provoke. Moreover, Edward Belbruno was the main organizer of six international conferences ”New Trends in Astrodynamics” sponsored by NASA. The proceedings of these conferences vividly illustrate the great impact the new theories and methods of Edward Belbruno has on the vast field of aerodynamics.

Although it is now well accepted that ballistic capture in transfers to the Moon works, people generally remained skeptical about the question if Hiten-like transfers, or, more generally, ballistic capture transfers, can also be applied successfully to Mars missions. Indeed, the orbital velocity of Mars is much higher than the one of the Earth’s Moon. Again it was Edward Belbruno who jointly with Francesco Topputo made the crucial breakthrough and showed in an impressive paper two years ago that low energy transfer is as well feasible in Mars missions [8]. These discoveries had a huge resonance in the world press. In Scientific American an article entitled “A New Way to Reach Mars Safely, Anytime and on the Cheap“ [9] enthusiastically reported about the new discoveries of Belbruno and Topputo and soon later Spiegel Online followed with an article “Zum Mars lupfen: Neue Flugroute zum Roten Planeten“ [10]. Indeed, the new methods of Belbruno and Topputo not only can safe a lot of fuel in future Mars missions they also provide a possibility to significantly increase the “launch window“ for missions to Mars. With traditional methods a rocket can only be launched to Mars if the Earth and Mars are in optimal orbital alignment. Via the new routes of Belbruno and Topputo one does not need to fly directly to Mars, but just close to Mars orbit, even if Mars is not there, and then get captured by the distant Mars.

The new opportunities the discoveries by Belbruno and Topputo provide presumably will be a major ingredient of the research of Edward Belbruno in the next years. This fits very well with theoretical progress which goes on currently in symplectic geometry and we expect a lot of interesting interactions between the technology of weak stability boundaries developed by Edward Belbruno and modern symplectic tools. A crucial problem in space mission design is how to reach a point B from a point A. Here the points A and B have to be interpreted as points in phase space, i.e., we do not just care about the position but as well about the velocity, since a change of the velocity requires a lot of fuel which is highly expensive to bring along. The new methology of Edward Belbruno allows to interpret these problems in terms of the language of modern symplectic geometry, namely as chords satisfying Lagrangian respectively Legendrian boundary conditions. Modern symplectic geometry developed homological counting tools for such chords like Floer homology, Symplectic Field theory or Fukaya A_∞-categories. A common feature of these gadgets is that one usually has large chain complexes with low homology. The generators of the chain complexes are the chords and the techniques of the weak stability boundary developed by Belbruno are the tools to locate the generators of these chain complexes.

In August 2016, a patent application was filed in the United States [11] for a new type of small spacecraft that is designed to take advantage of the types of trajectories and dynamics in regions where ballistic capture is possible, obtained from Belbruno's work. This spacecraft design is well suited for ballistic capture transfers to the Moon, Mars and other planets. This is a potentially important application to spacecraft design.

Apart from its practical relevance the new discoveries of Edward Belbruno in capture dynamics lead as well to new insights in theoretical astrophysics. With Richard Gott Edward Belbruno discussed in the paper "Where Did The Moon Come From" the origin of the Moon from the point of view of capture dynamics [12]. According to the Great Impact Theory the Moon originated from a crash of the Earth with a Mars-sized protoplanet. An open question in the Great Impact Theory is where the protoplanet came from. Belbruno and Gott provided a possible answer to this question by showing that if the protoplanet were at one of the equilateral Lagrange points ballistic capture could explain the collision of the protoplanet with the young Earth. Such a hypothetical planet is referred to as "Theia" according to the Greek goddess which gave birth to the Moon. We refer to [13] for information how NASA tried to detect remains of this ancient planet using operational spacecraft of the STEREO mission.

A different application of Edward Belbruno's Capture dynamics concerns the Lithopanspermia Hypothesis about the origin of life. According to this hypothesis microorganisms brought to Earth via meteorites were the sprouts of life on Earth. Jointly with Amaya Moro-Martín, Renu Malhotra and Dmitry Savransky the authors argue in the paper "Chaotic Exchange of Solid Material between Planetary Systems: Implications for Lithopanspermia" [14] that transfer of meteorites between two planetary systems could have been much more efficient than previously expected. In fact such a scenario could have occured elsewhere in the universe as well leading to life at planets different from Earth. "Slow moving rocks better odds that life crashed to Earth from space" reported Princeton University Press Release enthusiastically [15] and the new findings of Belbruno and his coworkers were mentioned on the cover of *Time Magazine* on October 22, 2012.

Before his involvement with NASA and mission design and even before getting his PhD already Edward Belbruno's masterthesis he wrote under the supervision of Juergen Moser had quite some impact. It is well known in Celestial mechanics that two body collisions can be regularized. Moser observed that for negative energy after regularization the Kepler flow becomes the geodesic flow on a sphere [16]. Edward Belbruno than showed that for zero energy the regularized Kepler flow becomes the geodesic flow in an Euclidean space and for positive energy it becomes the geodesic flow in Hyperbolic space [17, 18]. Surprisingly Edward Belbruno found in recent years quite striking applications of regularization techniques to rather different fields of physics. Jointly with Frans Pretorius he observed that Black holes can be regularized [19] and in [20] he applied regularization techniques to the big bang. An interesting question Edward Belbruno is researching at the moment is if this regularization reveals a KAM structure close to the big bang. The cosmological implications of such an additional structure of the big bang are waiting for being revealed.

A distinguishing feature of Edward Belbruno is his extremely broad spectrum of expertises. Trained as a pure mathematician, he worked a long time for NASA and consulted for Boeing, and other aerospace companies, while working as well with theoretical physicists and astrobiologists so that he cannot only talk to people on the theoretical side but to people with a deep understanding of applied problems. As such his work is extremely rare and valuable in bringing people from the theoretical and applied sides together. In view of his own methodology and his fantastic insight how to find interesting trajectories to design orbits, I expect from a visit of Edward Belbruno to Germany a boost with a longterm impact on theoretical as well as applied celestial mechanics.

2 Information on the invitation to Germany

My recent own research deals with the question how global methods of modern symplectic geometry like holomorphic curves and Floer homology can be applied to the restricted three body problem [21, 22]. In the restricted three body problem one considers two masses the primaries which attract a massless object referred to as the satellite via Newton's law of gravitation. Possible interpretation are that the two primaries are the Earth and the Moon or the Sun and Mars. If the satellite moves close to the heavy primary than one can consider the small primary as a perturbation which allows one to treat the restricted three body problem as a perturbation of the two problem which was solved already by Kepler. However, if the satellite moves close to the light primary, like the Mars or the Moon, perturbative methods are not applicable. In this case one has to apply new methods like holomorphic curves or Edwards Belbruno's Weak Stability Boundary techniques. Our shared research interest is therefore a global approach to the dynamics of the restricted three body problem.

Possible joint research projects are the following. It is knows that a bit above the first critical value the restricted three body problem has a periodic orbit in the neck between the two primaries which is called the Ljapunov orbit. The Ljapunov orbit is hyperbolic and therefore it has stable and unstable manifolds. A crucial question is if these manifolds intersect and if they do if they intersect transversally. Indeed, if this is the case this gives rise to homoclinic orbits and therefore to symbolic dynamics in the restricted three body problem. That for small mass ratios components of the stable and unstable manifold around the heavy primary intersect was shown by McGehee [23]. The transversality of the intersection was studied by Llibre, Martínez and Simó in [24], where as well numerical experiments were carried to see if the same holds true around the light primary. The project is now to use holomorphic curve techniques to show that the manifolds intersect around the heavy primary as well as around the light primary for mass ratios which are not necessarily small and then apply methods from the theory of the Weak Stability Boundary to prove transversality of the intersection. This research project as well requires a more thorough understanding between the relation of the Weak Stability Boundary and the stable manifold of the Ljapunov orbit about which important first research was carried out by Belbruno, Gidea, and Topputo in [25, 26].

A related research project is to understand the deviation of the stable and unstable manifolds of the Ljapunov orbit from the zero velocity curves. This question is motivated by fuel saving missions to Mars. Indeed, the zero velocity curves in this case roughly correspond to Mars orbit. In view of the relation between the stable manifold and the Weak Stability Boundary discussed above in case the stable manifold deviates much from the zero velocity curves, this gives possibilities to find orbits saving lot of fuel to Mars. This is because the satellite has not to have been sent as far as to Mars orbit in order to get captured by Mars. Pioneering steps in this direction were carried out by Belbruno and Topputo in [8].

A further application which should grow out of this research is a better understanding how much the launch window to Mars can be enlarged using capture techniques. Indeed, using traditional orbits based on Keplerian motion, a satellite to Mars can only be launched every 26 months, when Earth and Mars are on perfect alignment. As was pointed out by Belbruno and Topputo in [8] ballistic capture techniques can be used to enlarge the launch window. Indeed, the fact that one only needs to send the satellite to a point were it is captured by Mars, provides a lot more flexibility in the launch schedule for the satellite. At the moment how far the launch window can be increased using capture techniques is not known and it is part of the project to give precise estimates on this window, using refined techniques to estimate the Weak Stability Boundary.

Another interesting topic is to incorporate the ellipticity of Mars orbit into the theory. In fact if the two primaries in the restricted three body problem move around the common center of mass in circles the Jacobi integral is preserved. In case of elliptic orbits this is not anymore the case. Concerning the approach via holomorphic curves this requires to incorporate a time dependency in the complex structure which leads to interesting research questions in Symplectic Field Theory [27]. This is related to recent progress in Rabinowitz Floer homology [28, 29] because in the case of a time dependent Hamiltonian one needs to replace periodic orbits by discriminant points in the sense of Givental's nonlinear Maslov index. It is interesting to note that the notion of Weak Stability Boundary makes sense as well in the elliptic case. We expect from this research interesting relations to Aubry Mather theory. In particular, it should be possible to provide an interpretation of the Weak Stability Boundary as a kind of giant Aubry-Mather set.

Symplectic Geometry is very strong in Germany. This is as well vividly illustrated by the fact that the German Research Foundation (DFG) recently decided to fund the new Collaborative Research Centre (CRC) "Symplectic Structures in Geometry, Algebra and Dynamics". However, for the future development of Symplectic Geometry in Germany it is crucial to get a much more thorough understanding of the applied side of celestial mechanics. A longterm visit of Edward Belbruno would provide a fantastic opportunity to achieve this goal. It is hard to think of somebody more suitable for bringing people from the theoretical and applied side together than Edward Belbruno with his huge experience in both sides. Therefore Edward Belbruno plans as well to interact strongly with the people from the CRC, especially Prof. Peter Albers and Prof. Kai Zehmisch at the University of Münster, who are planning to organise conferences to bring people from the applied and theoretical side together and in which Edward Belbruno is supposed to play a crucial role.

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3 List of key publications of Edward Belbruno

- E. Belbruno, Lunar capture orbits, a method of constructing Earth-Moon trajectories and the lunar gas mission. In *Proceedings of AIAA/DGGLR/ JSASS Inter. Elec. Propl. Conf.*, number 87-1054, May 1987.

This is the first paper ever published on Ballistic Capture Transfers and Weak Stability Boundaries. In modern terminology the paper describes an Interior Weak Stability Transfer. In it Edward Belbruno shows that chaotic motion can be effectively used to find fuel preserving trajectories for a spacecraft from the Earth to the Moon. At this time people were very sceptical that this works in practice. However, when the first Japanese lunar mission was doomed to fail, J. Miller remembered this groundbreaking work of Edward Belbruno and together worked out with him a trajectory to safe the Hiten mission. The Interior Weak Stability Transfer discovered by Belbruno in this paper was later used in ESA's SMART-1 mission.

- E. Belbruno, J. Miller, A ballistic lunar capture trajectory for the Japanese spacecraft Hiten. Technical Report JPL-IOM 312/90.4-1731-EAB, Jet Propulsion Laboratory, June 15, 1990.

Although this is just a technical report it signifies the breakthrough which completely changed our understanding of space travel. The Hiten mission proved that Ballistic Capture Transfers already predicted before by Belbruno via his theory of the Weak Stability Boundary can be flown in practice. In the Hiten mission a four body problem is solved, involving the Earth, the Moon, the Sun as well as the spacecraft. This transfer Hiten used is now known as Exterior Weak Stability Transfer. It was used again, for example, in NASA's GRAIL mission in 2011.

- E. Belbruno, Capture Dynamics and Chaotic Motions in Celestial Mechanics: With Applications to the Construction of Low Energy Transfers, *Princeton University Press* (2004).

In this book Edward Belbruno explains on a high mathematical level the intricate dynamics which lies behind the saving of the Hiten mission. The book provides thorough definitions of different kinds of capture and the theory of the Weak Stability Boundary discovered by himself. Apart from that, the book contains a very readable account of celestial mechanics, dynamical systems and dynamical astronomy. A special feature is the very careful treatment of regularizations where Edward Belbruno explains in detail as well his own important contributions to the regularization of the Kepler problem for zero and positive energy. Therefore the book can also be used as an excellent starting point for the recent discoveries of Edward Belbruno about the regularization of black holes and the big bang.

- E. Belbruno, Fly Me to the Moon: An insider's Guide to the New Science of Space Travel, *Princeton University Press* (2007)

In this book Edward Belbruno describes for a broad audiance the paradigm change his work provided for space travel. The book has also a chapter on the theory of Belbruno and Gott about the origin of the Moon. In this theory they show that using Belbruno's theory of the Weak Stability Boundary it can be explained that a Mars-sized object, which originally stayed at one of the equilateral Lagrange points, could have collided with the Earth giving birth to the Moon.

- E. Belbruno ,M. Gidea, F. Topputo, Weak Stability Boundary and Invariant Manifolds, *SIAM J. Appl. Dyn. Syst. 9*, 1061-1089 (2010).

In this paper Belbruno, Gidea and Topputo describe the intricate geometrical structure of the Weak Stability Boundary and show how it is equivalent to the global manifolds associated to the Lyapunov orbits. This provided an important insight into the nature of these boundaries.

- E. Belbruno, F. Topputo, Earth-Mars Transfers With Ballistic Capture, *Celestial Mechanics and Dynamical Astronomy 121*, 329–346 (2015).

In this paper Belbruno and Topputo explain how using the theory of Belbruno of the Weak Stability Boundary ballistic capture orbits from the Earth to Mars can be found. This signifies a huge breakthrough for the future of ballistic capture. Indeed, the eccentricity of the orbit of the Mars is rather high and the orbital velocity of the Mars is much higher than the one of the Moon. The paper received an enthusiastic response in the world press like Scientific American or Spiegel Online.

4 List of Edward Belbruno's publications over the last ten years

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